

**Mathematics Standards**

**Statistics**

**Course Overview:** This course will include descriptive statistics for one-variable data and two-variable data, comparing one-variable samples, data collection, modeling one- and two-variable data, sample-to-sample variability, experimental and theoretical probability, counting principles, probability distributions, confidence intervals, sample size and an introduction to hypothesis testing.

**Bold standards are essential standards that all students will learn as they complete the course.**

**Unit 1: Describing One-Variable Data (approximately 12 days)**

**Description:** In this unit, students will use several methods for representing one-variable data, including dot plots, bar graphs, histograms, stem-and-leaf plots, and box plots and they will learn about the appropriate situations in which to use each of these representations. They will compute the mean, median, mode, range, interquartile range, and the mean absolute deviation for a data set and they will learn the vocabulary to help them to describe the shape of a data distribution. They will also learn about the characteristics of a statistical question.

**Standards**

1. Students will generate questions about data and explore ways to organize data to answer different questions. (S-ID.1)

2. **Students will analyze the strengths and weaknesses of various graphical representations of data.** (S-ID.1

3. Students will learn how to collect data and how to display the data in a stem-and-leaf plot and a dot plot. (S-ID.1)

4. Students will be introduced to measures of central tendency and will develop methods to find the mean, median, mode, and range of a set of data. (S-ID.2)

5. Students will compare two data sets using mean, median, mode, and range. (S-ID.2)

6. **Students will choose between median and mean to describe the “typical” middle value in a distribution of data.** (S-ID.3)

7. Students will develop a formula for the average distance from the mean in a set of data – the mean absolute deviation. (S-ID.2)

8. Students will construct and interpret box plots and compare sets of data. (S-ID.2)

9. Students will identify outliers using the 1.5 x IQR criteria. (S-ID.3)

10. **Students will determine the kinds of information that a representation communicates and decide which representation is the most useful for answering a specific question.** (S-ID.1, S-ID.2)

11. Students will identify statistical questions. They will understand that a statistical question is one that can be answered by collecting data and that they should expect variability in the answers. (S-IC.1)

**Unit 2: Intro to Probability (approximately 16 days)**

**Description:** In this unit, students will learn the vocabulary of probability and gain an understanding of the difference between experimental and theoretical probability. Students will analyze the consequences of changing the sample space and calculate compound probabilities. They will use a random number generator and computer simulations to estimate complex probabilities and they will consider multiple events and use strategies (lists, tables, trees, and area models) to represent all possible outcomes.

**Standards**

1. Students will develop an understanding that the probability of an event is always between 0 and 1. (S-CP.1)

2. Students will investigate theoretical versus experimental probabilities. (S-CP.1)

3. **Students will calculate theoretical probabilities.** (S-CP.1)

4. Students will investigate the role that the number of trials plays in the relationship between experimental and theoretical probabilities. (S-IC.2)

5. Students will describe what happens to the probability of an event when the sample space is changed by multiplication. (S-CP.1)

6. Students will calculate probabilities of “either-or” compound events. (S-CP.1, S-CP.7)

7. Students will apply strategies for calculating and comparing probabilities expressed as fractions, decimals, and percent’s. (S-CP.1)

8. Students will develop their understanding of uniform probability models, including the complement of an event. (S-CP.1)

9. Students will use computer simulation to find experimental probabilities of complex events. (S-IC.2)

10. Students will find probabilities of compound independent events and will determine whether pairs of events are dependent or independent. (S-CP.2)

11. Students will use probability tables to generate a complete list of possible outcomes of compound events and to calculate probabilities. (S-CP.2)

12. **Students will use tree diagrams to model outcomes for compound events.** (S-CP.2, S-CP.8 (+))

13. Students will calculate probabilities for multiple independent events where both outcomes are desired. (S-CP.2, S-CP.7)

14. Students will identify whether or not events are “mutually exclusive”. (S-CP.7)

15. **Students will use area diagrams to calculate probabilities.** (S-CP.2)

**Unit 3: Comparing One-Variable Samples** **(approximately 7 days)**

**Description:** In this unit, students will compare data distributions and they will quantify the overlap in two distributions using the IQR as a benchmark. They will learn that random samples are a good way to collect representative data. They will make inferences about the population from samples and they will informally determine intervals for which characteristics of the population fall.

**Standards**

1. **Students will compare sets of data by comparing their histograms and making parallel box plots to analyze center, shape, spread, and outliers.** (S-ID.2, S-ID.3)
2. Students will compare two populations based on making inferences from samples. (S-ID.2, S-IC.1)
3. Students will quantify the difference between the medians as a multiple of the interquartile range (IQR). (S-ID.3)
4. Students will analyze methods of sampling and critique how well a sample represents a certain population. (S-IC.3)
5. Students will use random sampling to draw inferences about a population. (S-IC.1)
6. Students will generate multiple samples of the same size to gauge the variation in sample statistics. (S-IC.2)

**Unit 4: Describing Two-Variable Data** (**approximately 10 days)**

**Description:** In this unit, students will create and interpret circle graphs representing one-variable categorical data and they will analyze two-variable numerical data with scatter plots. They will describe the form, direction, strength, and outliers for an association. Students will “eyeball” a line of best fit and use it to make predictions. Finally, students will return to categorical data and look for associations in two-way tables.

**Standards**

1. Students will use scatterplots for organizing data and making predictions. (S-ID.6)
2. Students will be introduced to the concept of dependent and independent variables and learn the importance of carefully scaling the axes of a graph. (S-ID.6)
3. Students will construct and interpret pie graphs using central angles and percents. (S-ID.1)
4. Students will create scatterplots and identify whether there is a relationship between the two sets of data. (S-ID.6)
5. Students will draw a line of best fit and use it to make predictions, when appropriate. (S-ID.7)
6. Students will categorize the strength and identify the direction of an association from a scatterplot. (S-ID.6)
7. Students will write the equation of for a line of best fit to represent scattered data that is roughly linear. (S-ID.7)
8. **Students will use their graph and their equation for a line of best fit to make and justify predictions.** (S-ID.7)
9. Students will interpret the slope and y-intercept of a line of best fit in context. (S-ID.7)
10. Students will describe an association between two numerical variables fully using form, direction, strength, and outliers. (S-ID.6)
11. Students will informally look for and describe associations between two categorical variables in two-way tables. (S-ID.5)
12. Students will develop an understanding that association can be seen in table rows or in table columns. (S-ID.5)

**Unit 5: Modeling One-Variable Data** **(approximately 14 days)**

**Description:** In this unit, students will discover that standard deviation is a better way to represent the spread of a data distribution. They will investigate how linear transformations affect the mean and standard deviation of a data set. They will investigate the normal density function and use the normal density function to compute percentiles.

**Standards**

1. Students will find and interpret standard deviation. (S-ID.2, S-ID.3)
2. Students will calculate sample standard deviations and perform linear transformations with one variable data. (S-ID.5)

3. Students will find and interpret z-scores. (S-ID.4)

4. Students will create relative frequency histograms that display percentages instead of counts. (S-ID.5)

5. **Students will model bell-shaped data with a normal curve and use the model to calculate proportions.** (S-ID.4)

6. Students will students will use normal curves to estimate make predictions about percentiles. (S-ID.4)

**Unit 6: Data Collection (approximately 12 days)**

**Description:** In this unit, students will focus on sources of data including censuses, observational studies, experiments, and computer simulations. They will learn that samples are used to represent populations, and how bias and lurking variables can unintentionally influence studies. The students will learn the different purposes of randomization in various types of studies and how randomization can make studies more valid. They will become more skeptical and analytical about studies they encounter in the media and thus become more informed consumers and citizens.

**Standards**

1. Students will compare and contrast samples and populations. (S-IC.1)

2. Students will identify bias in survey questions. (S-IC.6)

3. Students will understand the difference between open and closed questions. (S-IC.6)

4. Students will be introduced to the random sample as a method of choosing an unbiased sample that is representative of the population. (S-IC.1, S-IC.3)

5. **Students will analyze methods of sampling and develop techniques for trying to find a random and representative sample.** (S-IC.1, S-IC.4)

6. Students will collect and analyze sample data to make a statement about the population. (S-IC.1)

7. **Students will understand that observational studies are ineffective at finding cause and effect.** (S-ID.9)

8. Students will discover the importance of randomization in an experiment. (S-IC.3, S-IC.6)

**Unit 7: Modeling Two-Variable Data** **(approximately 12 days)**

**Description:** In this unit, students will model the relationship between two numerical variables using a line of best fit and they will interpret the slope and the y-intercept of the the model in the context of the problem. They will explore the limitations of models when extrapolated far from the edges of the actual data. The students will use their calculators to find the least squares regression line and to quantify the variability in data by finding correlation coefficients and coefficients of determination. Students will also look at residual plots to assess the fit of a linear model to their data and consider non-linear models when appropriate.

**Standards**

1. Students will calculate, interpret, and graphically represent a residual. (S-ID.6b)

2. Students will learn that extrapolation with a statistical model can lead to nonsensical results. (S-ID.7)

3. Students will graphically determine an upper and lower bound on the prediction they make from a linear best-fit model. (S-ID.6b)

4. **Students will find the least squares regression line (LSRL) on their calculator and understand that it is the line that minimizes the sum of the squares of the residuals.** (S-ID.6c)

5. Students will determine if a linear model is a good fit for the data by creating and visually analyzing residual plots. (S-ID.6b-c)

6. **Students will calculate the correlation coefficient, *r*, and observe the scatter for various extremes of *r*.** (S-ID.8)

7. Students will interpret the coefficient of determination, *r*2, in context. (S-ID.6, S-ID.7, S-ID.8)

8. Students will fit a non-linear model to data that shows a curved trend. (S-ID.6a-b)

**Unit 8: Sample-to-Sample Variability** **(approximately 11 days)**

**Description:** In this unit, students will learn that some complex probability problems are easier to solve using computer simulation than by trying to determine theoretical probabilities. They will use simulations to investigate “streaks” and may be surprised to find that “streaks” are much more common in probabilistic situations than they expect. Simulations will also be used to model natural sample-to-sample variability and place a margin of error on their estimates of population parameters and students will learn that a larger sample size will reduce the sample-to-sample variability in their model. Students will then get a preview of confidence intervals and hypothesis testing by using the results of a simulation of sample-to-sample variability to informally determine if the results of a survey are statistically significant.

**Standards**

1. Students will simulate a probability problem first by tossing coins and then by using the random number generator on their calculator. (S-IC.2)

2. Students will use technology to simulate the occurrence of streaks in flipping coins and also to simulate wait time for a particular event to occur. (S-IC.2)

3. **Students will us a physical simulation investigate the natural sample-to-sample variability and develop the idea of margin of error on a sample proportion when taking random samples from a population.** (S-IC.4)

4. Students will use a computer simulation to further investigate the natural sample-to-sample variability and determine the effect of sample size on margin of error. (S-IC.4)

5. Students will use a simulation to investigate whether a claim about a population based on survey data is plausible. (S-IC.4)

6. Students will simulate sample-to-sample variability to determine to determine if it is plausible that two treatments in an experiment are truly different. (S-IC.5)

7. Students will use simulations to decide whether or not a manufactured part is within typical quality specifications. (S-IC.5)

8. Students will simulate a quality control process. (S-MD.7 (+))

**Unit 9: Advanced Probability** **(approximately 16 days)**

**Description:** In this unit, students will learn to calculate probabilities of unions, and intersections of events and to improve efficiency for some probability calculations by using the complementary event. They will also calculate the expected value for games of chance. The students will explore conditional probability and independence and extend these ideas to determine probabilities and independence for data displayed in two-way tables.

**Standards**

1. Students will develop the concept of a fair game. (S-MD.5 (+))

2. Students will calculate probabilities of unions and intersections of events. (S-CP.1, S-CP.7)

3. Students will use complements as a strategy for efficiently calculating probabilities. (S-CP.1)

4. **Students will find the expected value of a game of chance.** (S-MD.2 (+))

5. Students will calculate conditional probabilities. (S-CP.3, S-CP.6)

6. Students will calculate probabilities from data arranged in two-way tables. (S-CP.4, S-CP.5, S-CP.6, S-CP.7, S-CP.8 (+))

7. **Students will determine if events are independent and if they are mutually exclusive.** (S-CP.2, S-CP.3, S-CP.4, S-CP.5)

8. Students will analyze the probability of winning and losing a game. They will collect experimental data and construct a probability model to represent the game. (S-MD.7 (+))

9. Students will solve a variety of real-life application problems that involve probability. (S-MD.5 (+), S-MD.6 (+), S-MD.7 (+))

**Unit 10: Counting Principles** **(approximately 10 days)**

**Description:** In this unit, students will develop systematic counting methods beginning with the Fundamental Counting Principal. They then extend this idea to develop formulas for permutations and combinations and they learn to recognize when a situation that can be counted as purely a permutation or a combination. The students then move on to more complicated counting problems for which they sometimes need to combine or modify these methods. They also use their counting techniques to solve challenging probability problems.

**Standards**

1. **Students will use the Fundamental Principle of Counting to count permutations and other outcomes when there are too many to list.** (S-CP.9 (+))

2. Students will identify permutations and develop two formulas for counting the number of permutations. (S-CP.9 (+))

3. Students will discover the difference between permutations and combinations. (S-CP.9 (+))

4. **Students will use their knowledge of counting permutations to develop a formula for counting combinations.** (S-CP.9 (+))

5. Students will determine counting methods for situations that involve order with repetition, order with no repetition, no order with repetition, and no order without repetition. (S-CP.9 (+))

6. Students will solve a variety of real-life application problems using counting techniques. (S-CP.9 (+))

**Unit 11: Probability Distributions** **(approximately 10 days)**

**Description:** In this unit, students will construct probability distributions for random variables, find the mean, variance, standard deviation, and expected value for a discrete random variables, and solve problems involving the binomial distribution.

**Standards**

1. Students will construct probability distributions for random variables. (S-MD.1 (+), S-MD.4 (+))

2. **Students will find the mean, variance, standard deviation, and expected value for discrete random variables.** (S-MD.2 (+))

3. Students will find the exact theoretical probability for *X* successes in *n* trials of a binomial experiment. (S-MD.3 (+))

4. Students will find the mean, variance, and standard deviation for the variable of a binomial distribution. (S-MD.3 (+))

**Unit 12: The Normal Distribution** **(approximately 14 days)**

**Description:** In this unit, students will learn about the properties of the normal distribution and use these ideas to solve application problems. They will learn about a very important idea regarding the normal distribution called the *Central Limit Theorem*. They will also use the normal distribution to as an approximation for the binomial distribution.

**Standards**

1. Students will identify various distributions as symmetric or skewed. (S-MD.1 (+))

2. Students will identify properties of the normal distribution. (S-MD.4 (+))

3. Students will find the area under the standard normal distribution, given various intervals of z-values. (S-MD.3 (+))

4. **Students will find probabilities for a normally distributed variable by transforming it into a standard normal variable.** (S-MD.3 (+))

5. Students will find specific data values for given percentages using the standard normal distribution. (S-MD.3 (+))

6. Students will use the Central Limit Theorem to solve problems involving sample means for large samples.

7. Students will use the normal approximation to compute probabilities for a binomial variable.

**Unit 13: Confidence Intervals and Sample Size** **(approximately 12 days)**

**Description:** In this unit, students will find the confidence interval for the mean when σ is known, find the confidence interval for the mean when σ is unknown, find the confidence interval and sample size for proportions, and find a confidence interval for a variance and a standard deviation.

**Standards**

1. Students will find the confidence interval for the mean when the population standard deviation is known.

2. Students will determine the minimum sample size for finding a confidence interval for the mean.

3. **Students will find the confidence interval for the mean when the population standard deviation deviation is unknown.**

4. Students will find the confidence interval for a proportion.

5. Students will determine the minimum sample size for finding a confidence interval for a proportion.

6. Students will find a confidence interval for a variance and a standard deviation.

**Unit 14: Hypothesis Testing** **(approximately 15 days)**

**Description:** In this unit, students will gain an understanding of the definitions and steps used in hypothesis testing and they will use the *z*-test for a mean, the *t*-test for a mean, the *z*-test for a proportion, and the *χ*2-test for a variance or a standard deviation.

**Standards**

1. Students will gain an understanding of the definitions used in hypothesis testing.

2. Students will state the null and alternative hypotheses.

3. Students will find critical values for the *z*-test.

4. Students will state the five steps used in hypothesis testing.

5. Students will test means when the population standard deviation is known using the *z*-test.

6. Students will test means when the population standard deviation is unknown using the *t*-test.

7. Students will test proportions using the *z*-test.

8. Students will test variances and standard deviations using the chi-square test.